



रोल नं.

Roll No.

प्रश्न-पत्र कोड

Q.P. Code

65/1/3

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.



गणित



MATHEMATICS

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

नोट

- (I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
- (II) कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
- (III) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- (IV) कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।
- (V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।

NOTE

- (I) Please check that this question paper contains 23 printed pages.
- (II) Please check that this question paper contains 38 questions.
- (III) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- (IV) Please write down the serial number of the question in the answer-book at the given place before attempting it.
- (V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- (i) इस प्रश्न-पत्र में कुल 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
- (ii) यह प्रश्न-पत्र पाँच खण्डों में विभाजित है – खण्ड-क, ख, ग, घ एवं ङ।
- (iii) खण्ड – क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं।
- (iv) खण्ड – ख में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं।
- (v) खण्ड – ग में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं।
- (vi) खण्ड – घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं।
- (vii) खण्ड – ङ में प्रश्न संख्या 36 से 38 प्रकरण अध्ययन आधारित 4 अंकों के प्रश्न हैं।
- (viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड – ख के 2 प्रश्नों में, खण्ड – ग के 3 प्रश्नों में, खण्ड – घ के 2 प्रश्नों में तथा खण्ड-ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
- (ix) कैल्कुलेटर का उपयोग वर्जित है।

खण्ड – क

इस खण्ड में 20 बहुविकल्पीय प्रश्न हैं। प्रत्येक प्रश्न का 1 अंक है।

20 × 1 = 20

1. एक रैखिक प्रोग्रामन समस्या, जिसका उद्देश्य फलन $Z = ax + by$ है, का सुसंगत क्षेत्र एक परिबद्ध क्षेत्र है, तो निम्न में से कौन सा सही है ?
 - (A) इसका केवल अधिकतम मान होगा।
 - (B) इसका केवल न्यूनतम मान होगा।
 - (C) इसके अधिकतम तथा न्यूनतम दोनों मान होंगे।
 - (D) इसका न तो अधिकतम और न ही न्यूनतम मान होगा।



General Instructions :

Read the following instructions very carefully and strictly follow them :

- (i) This Question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.
- (iii) In Section **A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) and Question Number **19 & 20** are Assertion-Reason based questions of **1** mark each.
- (iv) In Section **B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In Section **C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In Section **D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.
- (vii) In Section **E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, **2** questions in Section – **D** and **2** questions in Section – **E**.
- (ix) Use of calculator is **NOT** allowed.

SECTION – A

This section comprises of **20** Multiple Choice Questions (MCQs) of **1** mark each.

$$20 \times 1 = 20$$

1. If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct ?
 - (A) It will only have a maximum value.
 - (B) It will only have a minimum value.
 - (C) It will have both maximum and minimum values.
 - (D) It will have neither maximum nor minimum value.



2. सदिशों $\hat{i} - \hat{j}$ तथा $\hat{i} + \hat{j}$ दोनों के लंबवत मात्रक सदिश है

(A) $\hat{k}$

(B) $-\hat{k} + \hat{j}$

(C) $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$

(D) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

3. यदि $\int_0^1 \frac{e^x}{1+x} dx = \alpha$ है, तो $\int_0^1 \frac{e^x}{(1+x)^2} dx$ है

(A) $\alpha - 1 + \frac{e}{2}$

(B) $\alpha + 1 - \frac{e}{2}$

(C) $\alpha - 1 - \frac{e}{2}$

(D) $\alpha + 1 + \frac{e}{2}$

4. यदि $\int \frac{2^{\frac{1}{x}}}{x^2} dx = k \cdot 2^{\frac{1}{x}} + C$ है, तो k बराबर है

(A) $\frac{-1}{\log 2}$

(B) $-\log 2$

(C) -1

(D) $\frac{1}{2}$

5. यदि $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ है, तो A^{-1} है

(A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



2. The unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ is

(A) $\hat{k}$

(B) $-\hat{k} + \hat{j}$

(C) $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$

(D) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

3. If $\int_0^1 \frac{e^x}{1+x} dx = \alpha$, then $\int_0^1 \frac{e^x}{(1+x)^2} dx$ is equal to

(A) $\alpha - 1 + \frac{e}{2}$

(B) $\alpha + 1 - \frac{e}{2}$

(C) $\alpha - 1 - \frac{e}{2}$

(D) $\alpha + 1 + \frac{e}{2}$

4. If $\int \frac{2^x}{x^2} dx = k \cdot 2^{\frac{1}{x}} + C$, then k is equal to

(A) $\frac{-1}{\log 2}$

(B) $-\log 2$

(C) -1

(D) $\frac{1}{2}$

5. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is

(A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

6. यदि $\begin{bmatrix} x+y & 3y \\ 3x & x+3 \end{bmatrix} = \begin{bmatrix} 9 & 4x+y \\ x+6 & y \end{bmatrix}$ है, तो $(x-y)$ बराबर है

- (A) -7 (B) -3
(C) 3 (D) 7

7. माना M तथा N दो ऐसी घटनाएँ हैं कि $P(M) = 0.6$, $P(N) = 0.2$ तथा $P(M \cap N) = 0.5$ है, तो $P(M' / N')$ है

- (A) $\frac{7}{8}$ (B) $\frac{2}{5}$
(C) $\frac{1}{2}$ (D) $\frac{2}{3}$

8. निम्न में से कौन सा x तथा y में एक समघातीय फलन नहीं है ?

- (A) $y^2 - xy$ (B) $x - 3y$
(C) $\sin^2 \frac{y}{x} + \frac{y}{x}$ (D) $\tan x - \sec y$

9. यदि $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = \sqrt{37}$, $|\vec{b}| = 3$ तथा $|\vec{c}| = 4$ है, तो $\vec{b}$ तथा $\vec{c}$ के बीच का कोण है :

- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

10. यदि $f(x) = |x| + |x-1|$ है, तो निम्न में से कौन सा सही है ?

- (A) $x = 0$ तथा $x = 1$ पर $f(x)$ संतत तथा अवकलनीय है ।
(B) $x = 0$ तथा $x = 1$ पर $f(x)$ अवकलनीय है परन्तु संतत नहीं है ।
(C) $x = 0$ तथा $x = 1$ पर $f(x)$ संतत है परन्तु अवकलनीय नहीं है ।
(D) $x = 0$ तथा $x = 1$ पर $f(x)$ न तो संतत है और न ही अवकलनीय है ।

6. If $\begin{bmatrix} x+y & 3y \\ 3x & x+3 \end{bmatrix} = \begin{bmatrix} 9 & 4x+y \\ x+6 & y \end{bmatrix}$, then $(x-y) = ?$

(A) -7

(B) -3

(C) 3

(D) 7

7. Let M and N be two events such that $P(M) = 0.6$, $P(N) = 0.2$ and $P(M \cap N) = 0.5$, then $P(M'/N')$ is

(A) $\frac{7}{8}$

(B) $\frac{2}{5}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

8. Which of the following is not a homogeneous function of x and y ?

(A) $y^2 - xy$

(B) $x - 3y$

(C) $\sin^2 \frac{y}{x} + \frac{y}{x}$

(D) $\tan x - \sec y$

9. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = \sqrt{37}$, $|\vec{b}| = 3$ and $|\vec{c}| = 4$, then angle between $\vec{b}$ and $\vec{c}$ is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

10. If $f(x) = |x| + |x-1|$, then which of the following is correct ?

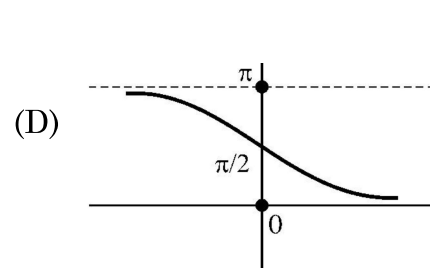
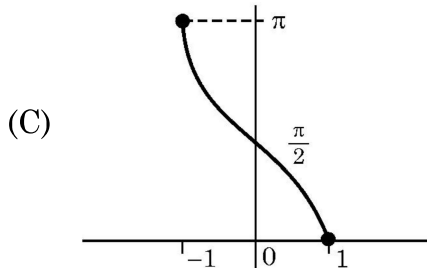
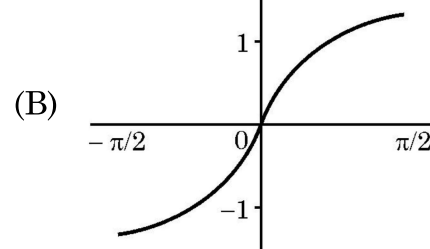
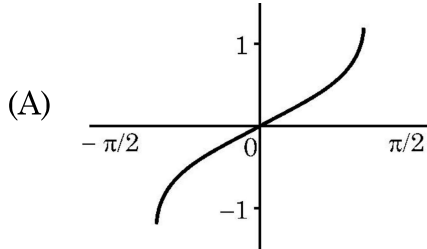
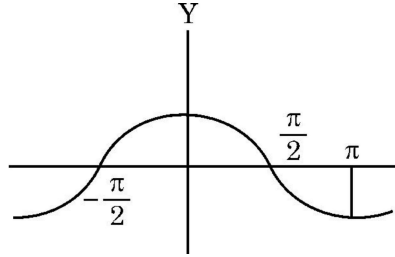
(A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$.

(B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$.

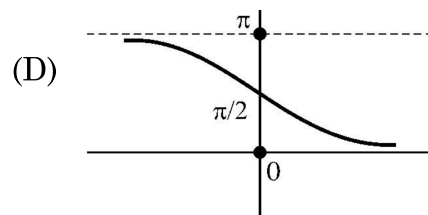
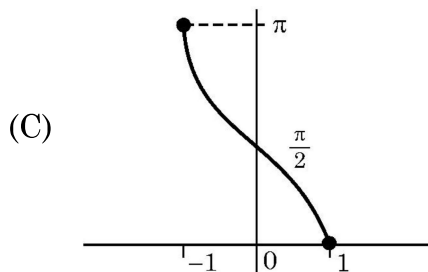
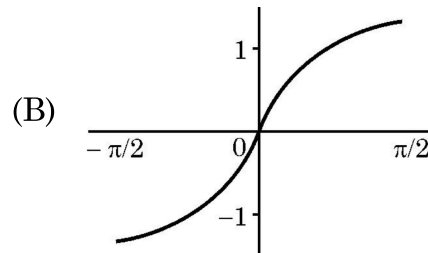
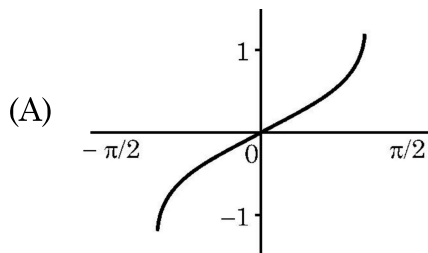
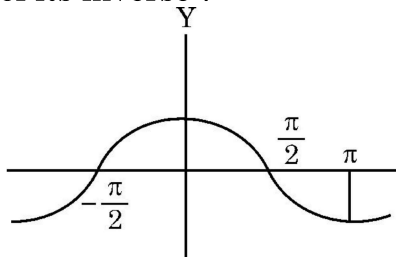
(C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.

(D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.

11. एक रैखिक समीकरण निकाय $AX = B$ से निरूपित है, जहाँ A गुणांकों का आव्यूह है, X चरों का आव्यूह है तथा B एक स्थिरांकों का आव्यूह है, तो समीकरण निकाय है
- (A) संगत है यदि $|A| \neq 0$ है तथा इसका हल $X = BA^{-1}$ द्वारा प्रदत्त है।
 (B) असंगत है यदि $|A| = 0$ तथा $(\text{adj } A) B = 0$ है।
 (C) असंगत है यदि $|A| \neq 0$ है।
 (D) संगत हो अथवा न हो यदि $|A| = 0$ तथा $(\text{adj } A) B = 0$
12. फलन $f(x) = x^3 - 3x + 2$ का $[0, 2]$ में निरपेक्ष उच्चतम मान है :
- (A) 0 (B) 2
 (C) 4 (D) 5
13. अवकल समीकरण $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^5 = \frac{d^2y}{dx^2}$ की कोटि तथा घात है :
- (A) कोटि 1, घात 1 (B) कोटि 1, घात 2
 (C) कोटि 2, घात 1 (D) कोटि 2, घात 2
14. एक त्रिकोणमितीय फलन का ग्राफ दिया गया है। निम्न में से कौन सा इसके व्युत्क्रम के ग्राफ को दर्शाता है ?



11. A system of linear equations is represented as $AX = B$, where A is coefficient matrix, X is variable matrix and B is the constant matrix. Then the system of equations is
- (A) Consistent, if $|A| \neq 0$, solution is given by $X = BA^{-1}$.
 (B) Inconsistent if $|A| = 0$ and $(\text{adj } A) B = 0$
 (C) Inconsistent if $|A| \neq 0$
 (D) May or may not be consistent if $|A| = 0$ and $(\text{adj } A) B = 0$
12. The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is :
- (A) 0 (B) 2
 (C) 4 (D) 5
13. The order and degree of differential function $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^5 = \frac{d^2y}{dx^2}$ are
- (A) order 1, degree 1 (B) order 1, degree 2
 (C) order 2, degree 1 (D) order 2, degree 2
14. The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse ?

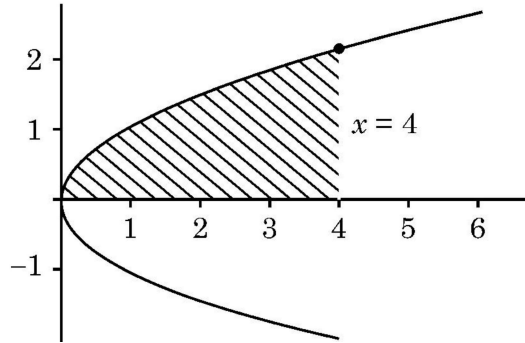


15. एक रैखिक प्रोग्रामिंग समस्या के आलेखीय निरूपण के सुसंगत क्षेत्र के कोनीय शीर्ष $(2, 72)$, $(15, 20)$ और $(40, 15)$ हैं। यदि $Z = 18x + 9y$ इसका उद्देश्य फलन है, तो
- (A) Z $(2, 72)$ पर अधिकतम, तथा $(15, 20)$ पर न्यूनतम होगा।
 (B) Z $(15, 20)$ पर अधिकतम, तथा $(40, 15)$ पर न्यूनतम होगा।
 (C) Z $(40, 15)$ पर अधिकतम, तथा $(15, 20)$ पर न्यूनतम होगा।
 (D) Z $(40, 15)$ पर अधिकतम, तथा $(2, 72)$ पर न्यूनतम होगा।

16. माना $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$ तथा $C = [9 \ 8 \ 7]$ है, तो निम्न में से कौन सा परिभाषित है ?
- (A) केवल AB (B) केवल AC
 (C) केवल BA (D) सभी AB, AC तथा BA

17. यदि A तथा B व्युत्क्रमणीय आव्यूह हैं, तो निम्न में से कौन सा सही नहीं है ?
- (A) $(A + B)^{-1} = B^{-1} + A^{-1}$ (B) $(AB)^{-1} = B^{-1}A^{-1}$
 (C) $\text{adj}(A) = |A| A^{-1}$ (D) $|A|^{-1} = |A^{-1}|$

18. वक्र $y^2 = x$, $x = 4$ तथा x -अक्ष के बीच के छायांकित क्षेत्र का क्षेत्रफल प्रदत्त है :



- (A) $\int_0^4 x \, dx$ (B) $\int_0^2 y^2 \, dy$
 (C) $2 \int_0^4 \sqrt{x} \, dx$ (D) $\int_0^4 \sqrt{x} \, dx$

15. The corner points of the feasible region in graphical representation of a L.P.P. are (2, 72), (15, 20) and (40, 15). If $Z = 18x + 9y$ be the objective function, then

- (A) Z is maximum at (2, 72), minimum at (15, 20)
 (B) Z is maximum at (15, 20) minimum at (40, 15)
 (C) Z is maximum at (40, 15), minimum at (15, 20)
 (D) Z is maximum at (40, 15), minimum at (2, 72)

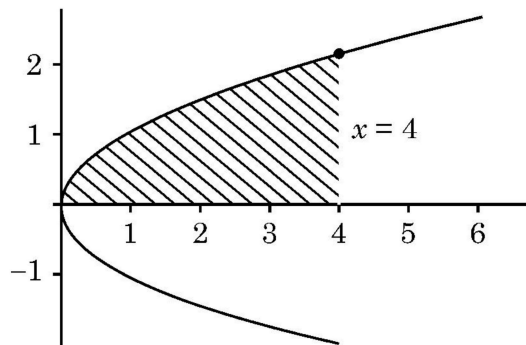
16. Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$, $C = [9 \ 8 \ 7]$, which of the following is defined ?

- (A) Only AB (B) Only AC
 (C) Only BA (D) All AB, AC and BA

17. If A and B are invertible matrices, then which of the following is not correct ?

- (A) $(A + B)^{-1} = B^{-1} + A^{-1}$ (B) $(AB)^{-1} = B^{-1}A^{-1}$
 (C) $\text{adj}(A) = |A| A^{-1}$ (D) $|A|^{-1} = |A^{-1}|$

18. The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x -axis is given by



- (A) $\int_0^4 x \, dx$ (B) $\int_0^2 y^2 \, dy$
 (C) $2 \int_0^4 \sqrt{x} \, dx$ (D) $\int_0^4 \sqrt{x} \, dx$

अभिकथन – तर्क आधारित प्रश्न

निर्देश : प्रश्न संख्या 19 और 20 अभिकथन (A) और तर्क (R) आधारित प्रश्न हैं, प्रत्येक प्रश्न का 1 गुण है। दो कथन दिए गए हैं। जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए।

- (A) दोनों, अभिकथन (A) तथा तर्क (R) सही हैं। तर्क (R), अभिकथन (A) की सही व्याख्या करता है।
 (B) दोनों, अभिकथन (A) तथा तर्क (R) सही हैं परंतु तर्क (R), अभिकथन (A) की सही व्याख्या नहीं करता है।
 (C) अभिकथन (A) सही है परन्तु तर्क (R) गलत है।
 (D) अभिकथन (A) गलत है जबकि तर्क (R) सही है।

19. अभिकथन (A) : $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$

$k = \frac{5}{2}$ के लिए, $x = 5$ पर संतत है।

तर्क (R) : $x = a$ पर फलन f संतत है यदि $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$

20. अभिकथन (A) : माना Z पूर्णाकों का समुच्चय है। एक फलन $f : Z \rightarrow Z$, जो $f(x) = 3x - 5$, $\forall x \in Z$ द्वारा परिभाषित है, एकैकी-आच्छादक है।

तर्क (R) : वह फलन जो एकैकी है तथा आच्छादक भी है, एकैकी-आच्छादक होता है।

खण्ड – ख

$5 \times 2 = 10$

इस खण्ड में 5 अति लघु-उत्तरीय प्रश्न हैं। प्रत्येक प्रश्न के 2 अंक हैं।

21. एक समांतर चतुर्भुज का क्षेत्रफल ज्ञात कीजिए जिसके विकर्ण $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ तथा $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$ द्वारा निरूपित हैं।

22. 'a' के वे मान ज्ञात कीजिए, जिनके लिए $f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$, $\mathbb{R}$ पर हासमान फलन हो।

Assertion – Reason Based Questions

Direction : Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$

is continuous at $x = 5$ for $k = \frac{5}{2}$.

Reason (R) : For a function f to be continuous at $x = a$,

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a).$$

20. **Assertion (A) :** Let Z be the set of integers. A function $f : Z \rightarrow Z$ defined as $f(x) = 3x - 5, \forall x \in Z$ is a bijective.

Reason (R) : A function is a bijective if it is both surjective and injective.

SECTION – B

5 × 2 = 10

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.

21. The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Find the area of the parallelogram.

22. Find the values of 'a' for which $f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$ is decreasing on $\mathbb{R}$.



23. (a) दो मित्र अलग-अलग स्थानों से पतंग उड़ाते समय पाते हैं कि उनकी पतंग की डोर एक दूसरे को छू रही हैं। डोरियों को सदिशों $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ तथा $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ द्वारा निरूपित किया जा सकता है। यह मानते हुए कि डोरियों में कोई ढील नहीं है, उनके बीच का कोण ज्ञात कीजिए।

अथवा

- (b) परिमाण 21 इकाई का एक सदिश ज्ञात कीजिए जो सदिश $\vec{AB}$ की विपरीत दिशा में हो जहाँ $A(2, 1, 3)$ तथा $B(8, -1, 0)$ हैं।

24. x के लिए हल कीजिए :

$$2 \tan^{-1} x + \sin^{-1} \left(\frac{2x}{1+x^2} \right) = 4\sqrt{3}$$

25. (a) $2^{\cos^2 x}$ का $\cos^2 x$ के सापेक्ष अवकलन कीजिए।

अथवा

- (b) यदि $\tan^{-1}(x^2 + y^2) = a^2$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

खण्ड – ग

6 × 3 = 18

इस खण्ड में 6 लघु उत्तरीय प्रश्न हैं। प्रत्येक प्रश्न के 3 अंक हैं।

26. निम्न रेखिक प्रोग्रामन समस्या को ग्राफ द्वारा हल कीजिए :

$$\text{व्यवरोधों } 2x + 3y \leq 6$$

$$3x - 2y \leq 6$$

$$y \leq 1, x \geq 0, y \geq 0$$

के अंतर्गत $Z = 8x + 9y$ का अधिकतमीकरण करना।

27. (a) ज्ञात कीजिए : $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$

अथवा

- (b) मान ज्ञात कीजिए : $\int_0^5 (|x-1| + |x-2| + |x-5|) dx$

23. (a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings.

OR

- (b) Find a vector of magnitude 21 units in the direction opposite to that of $\vec{AB}$ where A and B are the points A(2, 1, 3) and B(8, -1, 0) respectively.

24. Solve for x ,

$$2 \tan^{-1} x + \sin^{-1} \left(\frac{2x}{1+x^2} \right) = 4\sqrt{3}$$

25. (a) Differentiate $2^{\cos^2 x}$ w.r.t $\cos^2 x$.

OR

- (b) If $\tan^{-1} (x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.

SECTION – C

6 × 3 = 18

This section comprises of **6** Short Answer (SA) type questions of **3** marks each.

26. Solve the following linear programming problem graphically :

Maximize $Z = 8x + 9y$

Subject to the constraints :

$$2x + 3y \leq 6$$

$$3x - 2y \leq 6$$

$$y \leq 1$$

$$x \geq 0, y \geq 0$$

27. (a) Find : $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$

OR

- (b) Evaluate : $\int_0^5 (|x-1| + |x-2| + |x-5|) dx$



28. एक गोलाकार दवा का गोला जब पानी में डाला जाता है तो वह इस प्रकार घुल जाता है कि किसी भी क्षण आयतन में कमी की दर उसके पृष्ठीय क्षेत्रफल के समानुपाती होती है। इसकी त्रिज्या की कमी की दर की गणना करिए।

29. $y = |x + 3|$ का ग्राफ बनाइए तथा समाकलन के प्रयोग से वक्र तथा x -अक्ष के बीच के उस क्षेत्र का क्षेत्रफल ज्ञात कीजिए जो $x = -6$ तथा $x = 0$ के बीच में है।

30. (a) सत्यापित कीजिए कि $\vec{r} = (1 - \lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$ तथा $\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$ द्वारा प्रदत्त रेखाएँ विषमतलीय हैं। अतः इनके बीच की न्यूनतम दूरी ज्ञात कीजिए।

अथवा

(b) एक क्रिकेट मैच में, गेंदबाज, विकेट कीपर और लेग स्लिप क्षेत्र रक्षक की स्थिति एक पंक्ति में हैं तथा $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ तथा $\vec{F} = 12\hat{i} + 18\hat{j}$ द्वारा क्रमशः प्रदत्त है। वह अनुपात ज्ञात कीजिए जिसमें विकेट कीपर, गेंदबाज और लेग स्लिप क्षेत्र रक्षक को मिलाने वाले रेखा खण्ड को विभाजित करता है।

31. (a) किसी शनिवार, एक कक्षा में विद्यार्थियों की अनुपस्थिति की संख्या का प्रायिकता बंटन नीचे दिया गया है :

X	0	2	4	5
P(X)	p	2p	3p	p

जहाँ X, अनुपस्थित रहने वाले विद्यार्थियों की संख्या है।

(i) p का मान ज्ञात कीजिए।

1

(ii) शनिवार को अनुपस्थित रहने वाले विद्यार्थियों की संख्या का माध्य ज्ञात कीजिए।

2

अथवा

(b) समाचार पत्र में विज्ञापित रिक्तियों के लिए 3000 उम्मीदवारों ने आवेदन दिए। प्राप्त आँकड़ों से यह पाया गया कि कुल आवेदनकर्ताओं का $\frac{2}{3}$ भाग महिलाएँ थीं तथा शेष पुरुष थे। नौकरी के लिए चयन लिखित परीक्षा द्वारा किया गया। आवेदनकर्ताओं के प्रदर्शन से संकेत मिलता है कि लिखित परीक्षा में एक पुरुष के विशिष्ट अंक प्राप्त करने की प्रायिकता 0.4 है, तथा महिला के विशिष्ट अंक प्राप्त करने की प्रायिकता 0.35 है। प्रायिकता ज्ञात कीजिए कि यादृच्छया चुना गया आवेदक, लिखित परीक्षा में विशिष्ट अंक प्राप्त करने वाला है।

28. A spherical medicine ball when dropped in water dissolves in such a way that the rate of decrease of volume at any instant is proportional to its surface area. Calculate the rate of decrease of its radius.
29. Sketch the graph of $y = |x + 3|$ and find the area of the region enclosed by the curve, x -axis, between $x = -6$ and $x = 0$, using integration.
30. (a) Verify that lines given by $\vec{r} = (1 - \lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$ and $\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines.

OR

- (b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.
31. (a) The probability distribution for the number of students being absent in a class on a Saturday is as follows :

X	0	2	4	5
P(X)	p	2p	3p	p

Where X is the number of students absent.

- (i) Calculate p. 1
- (ii) Calculate the mean of the number of absent students on Saturday. 2

OR

- (b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.



इस खण्ड में 4 दीर्घ-उत्तरीय प्रश्न हैं। प्रत्येक प्रश्न के 5 अंक हैं।

32. एक स्कूल अपने विद्यार्थियों को निम्न शर्तों के साथ तीन क्लबों में आबंटित करना चाहता है : खेल, संगीत और नाटक :

- खेल क्लब के विद्यार्थियों की संख्या, संगीत तथा नाटक क्लबों के विद्यार्थियों की संख्या के योगफल के समान हो।
- संगीत क्लब के विद्यार्थियों की संख्या, खेल क्लब के विद्यार्थियों की संख्या के आधे से 20 अधिक हो।
- इन तीनों क्लबों में आबंटित विद्यार्थियों की कुल संख्या 180 हो।

मैट्रिक विधि द्वारा विभिन्न क्लबों में आबंटित विद्यार्थियों की संख्या ज्ञात कीजिए।

33. ज्ञात कीजिए : $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$

34. (a) यदि $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ है, तो सिद्ध कीजिए कि $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

अथवा

(b) यदि $x = a \left(\cos \theta + \log \tan \frac{\theta}{2} \right)$ तथा $y = \sin \theta$ है, तो $\theta = \frac{\pi}{4}$ पर $\frac{d^2y}{dx^2}$ ज्ञात कीजिए।

35. (a) रेखा $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ में बिंदु A(1, 6, 3) का प्रतिबिंब A' ज्ञात कीजिए। बिंदुओं A तथा A' को मिलाने वाली रेखा का समीकरण भी ज्ञात कीजिए।

अथवा

(b) रेखा $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ पर एक ऐसा बिंदु P ज्ञात कीजिए जिसकी बिंदु Q(2, 4, -1) से दूरी 7 इकाई हो। P तथा Q को मिलाने वाली रेखा का समीकरण भी ज्ञात कीजिए।

This section comprises of 4 Long Answer (LA) type questions of 5 marks each.

32. A school wants to allocate students into three clubs : Sports, Music and Drama, under following conditions :

- The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club.
- The number of students in Music club should be 20 more than half the number of students in Sports club.
- The total number of students to be allocated in all three clubs are 180.

Find the number of students allocated to different clubs, using matrix method.

33. Find : $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$.

34. (a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

(b) If $x = a \left(\cos \theta + \log \tan \frac{\theta}{2} \right)$ and $y = \sin \theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

35. (a) Find the image A' of the point A(1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

Also, find the equation of the line joining A and A'.

OR

(b) Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point Q(2, 4, -1) is 7 units. Also, find the equation of line joining P and Q.

इस खण्ड में 3 प्रकरण आधारित प्रश्न हैं। प्रत्येक प्रश्न के 4 अंक हैं।

3 × 4 = 12

36. एक कक्षा शिक्षक अपने छात्रों द्वारा उन्हें सिखाई गई “संबंधों” की अवधारणा की सीख का आकलन करने के लिए उत्सुक है। वह निम्न पाँच संबंध जिनमें प्रत्येक समुच्चय $A = \{1, 2, 3\}$ पर परिभाषित है, लिखती है :

$$R_1 = \{(2, 3), (3, 2)\}$$

$$R_2 = \{(1, 2), (1, 3), (3, 2)\}$$

$$R_3 = \{(1, 2), (2, 1), (1, 1)\}$$

$$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$$

$$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$$

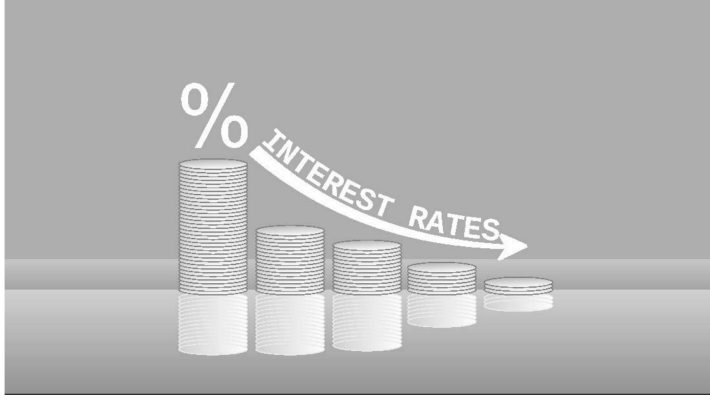
विद्यार्थियों से उपरोक्त संबंधों के लिए निम्न लिखे प्रश्नों का उत्तर देने के लिए कहा जाता है :

- संबंध ज्ञात कीजिए जो स्वतुल्य और संक्रामक है परन्तु सममित नहीं है।
- संबंध ज्ञात कीजिए जो स्वतुल्य और सममित है परन्तु संक्रामक नहीं है।
- (a) संबंध ज्ञात कीजिए जो सममित हैं पर न तो स्वतुल्य और न ही संक्रामक है।

अथवा

- (b) संबंध R_2 को तुल्यता संबंध बनाने के लिए जिन युग्मों को जोड़ना होगा, उन्हें लिखिए।

37.



एक बैंक अपने ग्राहकों को विभिन्न प्रकार के ब्याज पर ऋण उपलब्ध कराता है, जैसे कि फिक्स्ड दर, फ्लोटिंग दर और वेरिएबल दर। बैंक के पिछले आँकड़ों से यह ज्ञात है कि ग्राहक फिक्स्ड दर, फ्लोटिंग दर या वेरिएबल दर पर क्रमशः 10%, 20% और 70% प्रायिकता के साथ ऋण प्राप्त करते हैं। ऋण लेने के बाद ग्राहक ऋण का भुगतान कर सकता है या ऋण चुकाने में चूक भी कर सकता है। बैंक के आँकड़ों से पता चलता है कि फिक्स्ड रेट, फ्लोटिंग रेट और वेरिएबल रेट पर ऋण लेने के बाद किसी व्यक्ति द्वारा ऋण भुगतान पर चूक करने की प्रायिकता क्रमशः 5%, 3% और 1% है।

उपरोक्त के आधार पर निम्न के उत्तर दीजिए :

- ऋण लेने के बाद एक ग्राहक के ऋण चुकाने में चूक करने की क्या प्रायिकता है ? 2
- एक ग्राहक ऋण लेने के बाद चुकाने में चूक करता है। क्या प्रायिकता है कि उसने वेरिएबल दर ब्याज पर ऋण लिया था ? 2

36. This section comprises of 3 case study/passage based questions of 4 marks each.
A class-room teacher is keen to assess the learning of her students the concept of “relations” taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$:

$$R_1 = \{(2, 3), (3, 2)\}$$

$$R_2 = \{(1, 2), (1, 3), (3, 2)\}$$

$$R_3 = \{(1, 2), (2, 1), (1, 1)\}$$

$$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$$

$$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$$

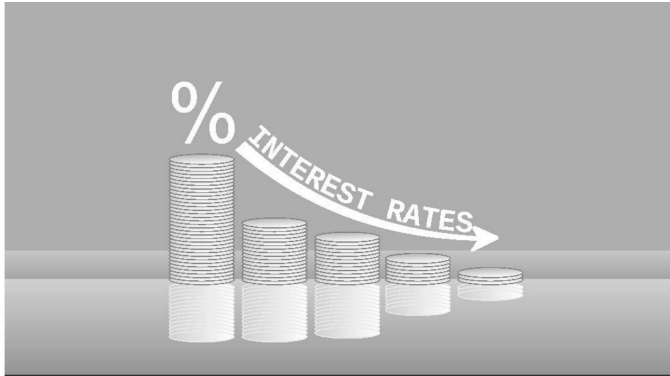
The students are asked to answer the following questions about the above relations :

- (i) Identify the relation which is reflexive, transitive but not symmetric.
- (ii) Identify the relation which is reflexive and symmetric but not transitive.
- (iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive.

OR

- (iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation ?

37.

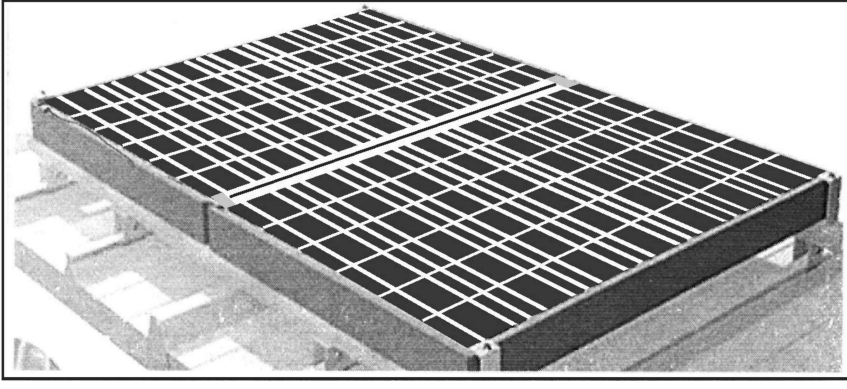


A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively.

Based on the above information, answer the following :

- (i) What is the probability that a customer after availing the loan will default on the loan repayment ? 2
- (ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest ? 2

38.



एक टेक्निकल कम्पनी आयताकार सौर पैनल बना रही है। जिन्हें छत पर लगाने के लिए उसके पास 300 मीटर सीमा पर लगाने की सामग्री है। पैनल के डिजाइन में एक साइड के समांतर विभाजन है जो पैनल को दो भागों में बाँटता है। माना विभाजन के लंबवत वाली भुजा की लंबाई x मीटर तथा इसके समांतर वाली भुजा की लंबाई y मीटर है।

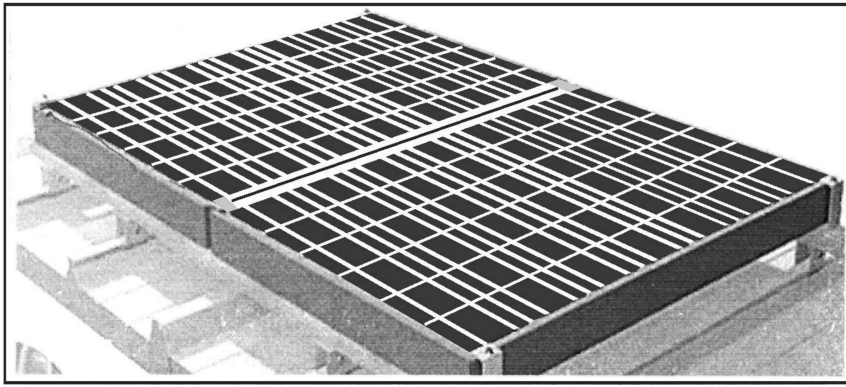
उपरोक्त सूचना के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

- (i) सीमा की कुल सामग्री जो सीमा तथा समांतर विभाजन में प्रयोग होनी है, के प्रयोग से x तथा y में बनी समीकरण ज्ञात कीजिए। 1
- (ii) सौर पैनल का क्षेत्रफल x के फलन के रूप में लिखिए। 1
- (iii) (a) क्षेत्रफल फलन के क्रांतिक बिंदु ज्ञात कीजिए। द्वितीय अवकलज के प्रयोग से वह क्रांतिक बिंदु ज्ञात कीजिए जिस पर क्षेत्रफल अधिकतम है। अधिकतम क्षेत्रफल भी ज्ञात कीजिए। 2

अथवा

- (iii) (b) प्रथम अवकलज परीक्षण के प्रयोग से 300 मीटर सीमा सामग्री के प्रयोग से परिबद्ध अधिकतम क्षेत्र का क्षेत्रफल ज्ञात कीजिए जहाँ समांतर विभाजन को भी लिया गया है। 2

38.



A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.

Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres.

Based on this information, answer the following questions :

- (i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y . 1
- (ii) Write the area of the solar panel as a function of x . 1
- (iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area. 2

OR

- (iii) (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition. 2



Marking Scheme Strictly Confidential (For Internal and Restricted use only) Senior Secondary Examination, 2025 SUBJECT NAME MATHEMATICS (Q.P. CODE – 65/1/3)	
<u>General Instructions: -</u>	
1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC.”
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (✓) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. This is the most common mistake which evaluators are committing.
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note “Extra Question” .

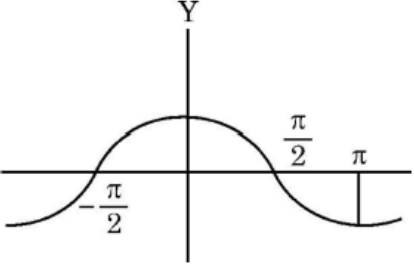
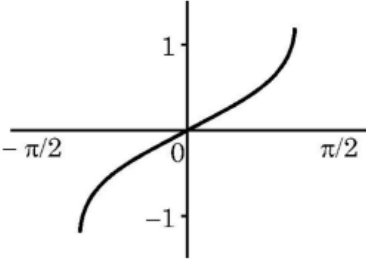
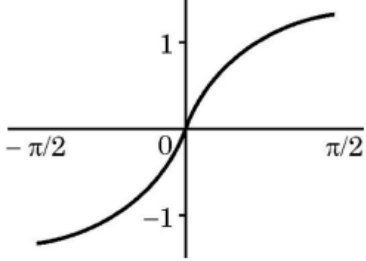
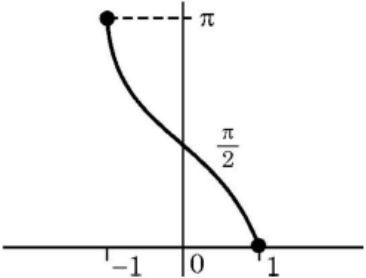
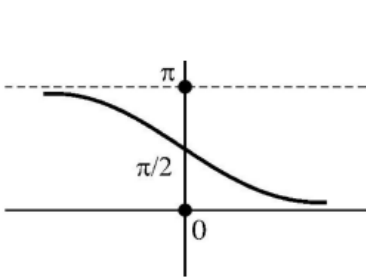
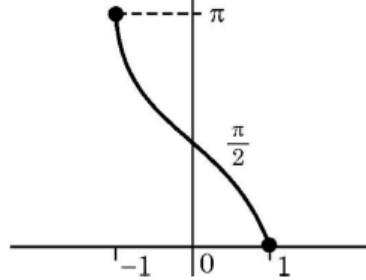
10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks _____ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - • Leaving answer or part thereof unassessed in an answer book. • Giving more marks for an answer than assigned to it. • Wrong totaling of marks awarded on an answer. • Wrong transfer of marks from the inside pages of the answer book to the title page. • Wrong question wise totaling on the title page. • Wrong totaling of marks of the two columns on the title page. • Wrong grand total. • Marks in words and figures not tallying/not same. • Wrong transfer of marks from the answer book to online award list. • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) • Half or a part of the answer marked correct and the rest as wrong, but no marks
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the “ Guidelines for Spot Evaluation ” before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

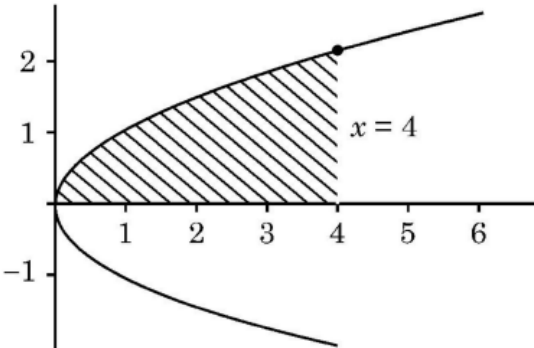
MARKING SCHEME
SENIOR SECONDARY EXAMINATION 2024-25
MATHEMATICS (Code-041)
[Paper Code: 65/1/3]

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION - A	
	Questions no. 1 to 18 are multiple choice questions (MCQs) of 1 mark each.	
Q1.	If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct ? (A) It will only have a maximum value. (B) It will only have a minimum value. (C) It will have both maximum and minimum values. (D) It will have neither maximum nor minimum value.	
Ans.	(C) It will have both maximum and minimum values.	1
Q2.	The unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ is (A) $\hat{k}$ (B) $-\hat{k} + \hat{j}$ (C) $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$	
Ans.	(A) $\hat{k}$	1
Q3.	If $\int_0^1 \frac{e^x}{1+x} dx = \alpha$, then $\int_0^1 \frac{e^x}{(1+x)^2} dx$ is equal to (A) $\alpha - 1 + \frac{e}{2}$ (B) $\alpha + 1 - \frac{e}{2}$ (C) $\alpha - 1 - \frac{e}{2}$ (D) $\alpha + 1 + \frac{e}{2}$	
Ans.	(B) $\alpha + 1 - \frac{e}{2}$	1

Q4.	If $\int \frac{2^x}{x^2} dx = k \cdot 2^{\frac{1}{x}} + C$, then k is equal to (A) $\frac{-1}{\log 2}$ (B) $-\log 2$ (C) -1 (D) $\frac{1}{2}$	
Ans	(A) $\frac{-1}{\log 2}$	1
Q5.	If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is (A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	
Ans	(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	1
Q6.	If $\begin{bmatrix} x+y & 3y \\ 3x & x+3 \end{bmatrix} = \begin{bmatrix} 9 & 4x+y \\ x+6 & y \end{bmatrix}$, then $(x-y) = ?$ (A) -7 (B) -3 (C) 3 (D) 7	
A6.	(B) -3	1
Q7.	Let M and N be two events such that $P(M) = 0.6$, $P(N) = 0.2$ and $P(M \cap N) = 0.5$, then $P(M'/N')$ is (A) $\frac{7}{8}$ (B) $\frac{2}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$	
Ans	(A) $\frac{7}{8}$	1

Q8.	Which of the following is <u>not</u> a homogeneous function of x and y ? (A) $y^2 - xy$ (C) $\sin^2 \frac{y}{x} + \frac{y}{x}$ (B) $x - 3y$ (D) $\tan x - \sec y$	
Ans	(D) $\tan x - \sec y$	1
Q9.	If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $ \vec{a} = \sqrt{37}$, $ \vec{b} = 3$ and $ \vec{c} = 4$, then angle between $\vec{b}$ and $\vec{c}$ is (A) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$	
Ans	(C) $\frac{\pi}{3}$	1
Q10.	If $f(x) = x + x - 1 $, then which of the following is correct ? (A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$. (B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$. (C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$. (D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.	
Ans	(C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.	1
Q11.	A system of linear equations is represented as $AX = B$, where A is coefficient matrix, X is variable matrix and B is the constant matrix. Then the system of equations is (A) Consistent, if $ A \neq 0$, solution is given by $X = BA^{-1}$. (B) Inconsistent if $ A = 0$ and $(\text{adj } A) B = 0$ (C) Inconsistent if $ A \neq 0$ (D) May or may not be consistent if $ A = 0$ and $(\text{adj } A) B = 0$	
Ans	(D) May or may not be consistent if $ A = 0$ and $(\text{adj } A) B = 0$	1
Q12.	The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is : (A) 0 (C) 4 (B) 2 (D) 5	
Ans	(C) 4	1

Q13.	The order and degree of differential function $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^5 = \frac{d^2y}{dx^2}$ are (A) order 1, degree 1 (B) order 1, degree 2 (C) order 2, degree 1 (D) order 2, degree 2	
Ans	(C) order 2, degree 1	1
Q14.	The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse ?  (A)  (B)  (C)  (D) 	
Ans	(C) 	1
Q15.	The corner points of the feasible region in graphical representation of a L.P.P. are (2, 72), (15, 20) and (40, 15). If $Z = 18x + 9y$ be the objective function, then (A) Z is maximum at (2, 72), minimum at (15, 20) (B) Z is maximum at (15, 20) minimum at (40, 15) (C) Z is maximum at (40, 15), minimum at (15, 20) (D) Z is maximum at (40, 15), minimum at (2, 72)	
Ans	(C) Z is maximum at (40, 15), minimum at (15, 20)	1

Q16.	Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$, $C = [9 \ 8 \ 7]$, which of the following is defined ? (A) Only AB (B) Only AC (C) Only BA (D) All AB, AC and BA	
Ans	(A) Only AB	1
Q17.	If A and B are invertible matrices, then which of the following is <u>not</u> correct ? (A) $(A + B)^{-1} = B^{-1} + A^{-1}$ (B) $(AB)^{-1} = B^{-1}A^{-1}$ (C) $\text{adj}(A) = A A^{-1}$ (D) $ A ^{-1} = A^{-1} $	
Ans	(A) $(A + B)^{-1} = B^{-1} + A^{-1}$	1
Q18.	The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x-axis is given by  (A) $\int_0^4 x \, dx$ (B) $\int_0^2 y^2 \, dy$ (C) $2 \int_0^4 \sqrt{x} \, dx$ (D) $\int_0^4 \sqrt{x} \, dx$	
Ans	(D) $\int_0^4 \sqrt{x} \, dx$	1

Assertion – Reason Based Questions

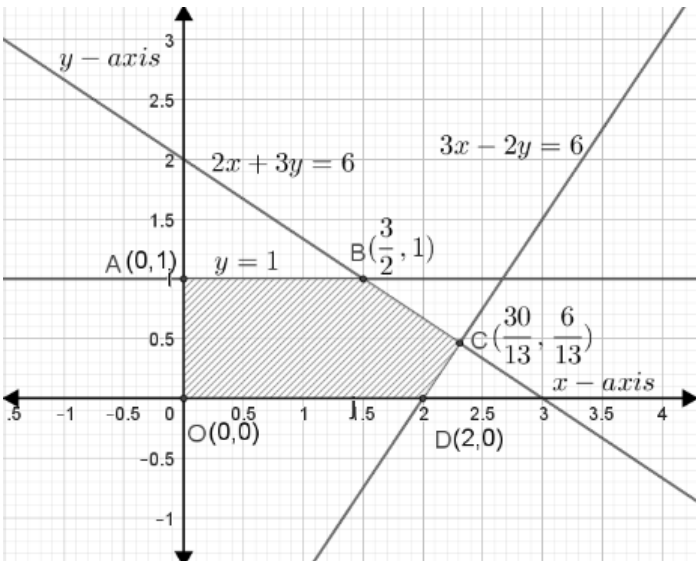
Direction : Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

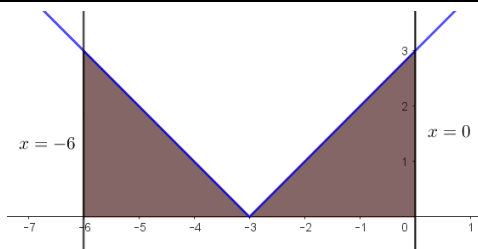
- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

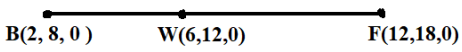
Q19.	Assertion (A) : $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$. Reason (R) : For a function f to be continuous at $x = a$, $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a).$	
Ans	(D) Assertion (A) is false, but Reason (R) is true.	1
Q20.	Assertion (A) : Let Z be the set of integers. A function $f : Z \rightarrow Z$ defined as $f(x) = 3x - 5, \forall x \in Z$ is a bijective. Reason (R) : A function is a bijective if it is both surjective and injective.	
Ans	(D) Assertion (A) is false, but Reason (R) is true.	1
SECTION B		
This section comprises very short answer (VSA) type questions of 2 marks each.		
Q21.	The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Find the area of the parallelogram.	
Ans	$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} = -2\hat{i} + 3\hat{j} + 7\hat{k}$ Area of parallelogram $= \frac{1}{2} \vec{a} \times \vec{b} $ $= \frac{1}{2} \sqrt{(-2)^2 + 3^2 + 7^2} = \frac{\sqrt{62}}{2}$	1
Q22.	Find the values of 'a' for which $f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$ is decreasing on $\mathbb{R}$.	
A22.	'f' is a decreasing function iff $f'(x) \leq 0 \Rightarrow \sqrt{3} \cos x + \sin x - 2a \leq 0$ $\Rightarrow \sin\left(x + \frac{\pi}{3}\right) \leq a$ We know that for all $x \in R, -1 \leq \sin\left(x + \frac{\pi}{3}\right) \leq 1$ $\Rightarrow a \in [1, \infty)$ or $(-\infty, -1]$	1 $\frac{1}{2}$ $\frac{1}{2}$

Q23.	(a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings. OR (b) Find a vector of magnitude 21 units in the direction opposite to that of $\vec{AB}$ where A and B are the points A(2, 1, 3) and B(8, -1, 0) respectively.	
Ans(a)	Let the required angle between the kite strings be θ. Then, $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{ \vec{a} \vec{b} }$ $\Rightarrow \cos\theta = \frac{(3\hat{i} + \hat{j} + 2\hat{k})(2\hat{i} - 2\hat{j} + 4\hat{k})}{\sqrt{9+1+4}\sqrt{4+4+16}} = \frac{12}{\sqrt{336}} = \frac{3}{\sqrt{21}}$ $\Rightarrow \theta = \cos^{-1}\left(\frac{12}{\sqrt{336}}\right) \text{ or } \cos^{-1}\left(\frac{3}{\sqrt{21}}\right)$	1½ ½
OR		
Ans(b)	$\vec{BA} = -6\hat{i} + 2\hat{j} + 3\hat{k}$ Required unit vector of magnitude 21 $= 21 \times \left(\frac{-6\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{36+4+9}} \right)$ $= 3(-6\hat{i} + 2\hat{j} + 3\hat{k}) \text{ or } -18\hat{i} + 6\hat{j} + 9\hat{k}$	1 ½ ½
Q24.	Solve for x, $2 \tan^{-1} x + \sin^{-1} \left(\frac{2x}{1+x^2} \right) = 4\sqrt{3}$	
Ans	$2 \tan^{-1} x + \sin^{-1} \left(\frac{2x}{1+x^2} \right) = 4\sqrt{3} \Rightarrow 2 \tan^{-1} x + 2 \tan^{-1} x = 4\sqrt{3}$ $\Rightarrow \tan^{-1} x = \sqrt{3} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$ $\therefore x = \tan(\sqrt{3})$ which has no solution.	1 1

Q25.	(a) Differentiate $2^{\cos^2 x}$ w.r.t $\cos^2 x$. OR (b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.	
Ans(a)	Let $u = 2^{\cos^2 x} \Rightarrow \frac{du}{dx} = 2^{\cos^2 x} (-2 \cos x \sin x) \log 2$ Let $v = \cos^2 x \Rightarrow \frac{dv}{dx} = -2 \cos x \sin x$ Now $\frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = 2^{\cos^2 x} \log 2$	1 $\frac{1}{2}$ $\frac{1}{2}$
OR		
Ans(b)	$\tan^{-1}(x^2 + y^2) = a^2 \Rightarrow x^2 + y^2 = \tan a^2$ Differentiate both sides wrt x, $2x + 2y \frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$	$\frac{1}{2}$ 1 $\frac{1}{2}$
SECTION C		
This section comprises short answer (SA) type questions of 3 marks each.		
Q26.	Solve the following linear programming problem graphically : Maximize $Z = 8x + 9y$ Subject to the constraints : $2x + 3y \leq 6$ $3x - 2y \leq 6$ $y \leq 1$ $x \geq 0, y \geq 0$	

Ans	 <table><tr><th>Corner Point</th><th>Value of $Z = 8x + 9y$</th></tr><tr><td>$A(0,1)$</td><td>9</td></tr><tr><td>$B\left(\frac{3}{2},1\right)$</td><td>21</td></tr><tr><td>$C\left(\frac{30}{13},\frac{6}{13}\right)$</td><td>$\frac{294}{13}$</td></tr><tr><td>$D(2,0)$</td><td>16</td></tr><tr><td>$O(0,0)$</td><td>0</td></tr></table> $\text{Max}(Z) = \frac{294}{13}$	Corner Point	Value of $Z = 8x + 9y$	$A(0,1)$	9	$B\left(\frac{3}{2},1\right)$	21	$C\left(\frac{30}{13},\frac{6}{13}\right)$	$\frac{294}{13}$	$D(2,0)$	16	$O(0,0)$	0	For correct graph and shading $1\frac{1}{2}$ For correct table 1 $\frac{1}{2}$
Corner Point	Value of $Z = 8x + 9y$													
$A(0,1)$	9													
$B\left(\frac{3}{2},1\right)$	21													
$C\left(\frac{30}{13},\frac{6}{13}\right)$	$\frac{294}{13}$													
$D(2,0)$	16													
$O(0,0)$	0													
Q27.	(a) Find : $\int \frac{2x-1}{(x-1)(x+2)(x-3)} \, dx$ OR (b) Evaluate : $\int_0^5 (x-1 + x-2 + x-5) \, dx$													
Ans(a)	$\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx = -\frac{1}{6} \int \frac{1}{x-1} dx - \frac{1}{3} \int \frac{1}{x+2} dx + \frac{1}{2} \int \frac{1}{x-3} dx$ (Using Partial Fraction) $= -\frac{1}{6} \log x-1 - \frac{1}{3} \log x+2 + \frac{1}{2} \log x-3 + C$	$1\frac{1}{2}$ $1\frac{1}{2}$												
OR														

Ans(b)	$I = \int_0^5 (x-1 + x-2 + x-5) dx$ $\therefore I = \left[-\int_0^1 (x-1) dx + \int_1^5 (x-1) dx \right] + \left[-\int_0^2 (x-2) dx + \int_2^5 (x-2) dx \right] + \left[-\int_0^5 (x-5) dx \right]$ $= -\frac{(x-1)^2}{2} \Big _0^1 + \frac{(x-1)^2}{2} \Big _1^5 - \frac{(x-2)^2}{2} \Big _0^2 + \frac{(x-2)^2}{2} \Big _2^5 - \frac{(x-5)^2}{2} \Big _0^5$ $= \frac{17}{2} + \frac{13}{2} + \frac{25}{2} = \frac{55}{2}$	$1\frac{1}{2}$ 1 $\frac{1}{2}$
Q28.	A spherical medicine ball when dropped in water dissolves in such a way that the rate of decrease of volume at any instant is proportional to its surface area. Calculate the rate of decrease of its radius.	
Ans	Let 'V' and 'S' be the volume and surface area of the spherical medicine ball with radius 'r'. $\frac{dV}{dt} = -kS, k > 0$ $V = \frac{4}{3} \pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow -kS = 4\pi r^2 \frac{dr}{dt}$ $\Rightarrow -k(4\pi r^2) = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = -k$ $\therefore$ Radius decreases at a constant rate.	1 $1 + \frac{1}{2}$ $\frac{1}{2}$
Q29.	Sketch the graph of $y = x + 3 $ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.	
Ans	Required Area $= \int_{-6}^0 y dx$ $= 2 \int_{-3}^0 (x+3) dx$ $= 2 \left[\frac{(x+3)^2}{2} \right]_{-3}^0$ $= 9$	 For correct graph: 1 mark $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

Q30.	(a) Verify that lines given by $\vec{r} = (1 - \lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$ and $\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines. OR (b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.	
Ans (a)	Rewriting the lines, we get $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$ Let $\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{a}_2 = \hat{i} - \hat{j} - \hat{k}$, $\vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$ Note that the dr's of given lines are not proportional so, they are not parallel lines. The lines will be skew if they do not intersect each other also. Here $\vec{a}_2 - \vec{a}_1 = \hat{j} - 4\hat{k}$, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{i} - 4\hat{j} - 3\hat{k}$ Consider $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$ $= (\hat{j} - 4\hat{k}) \cdot (2\hat{i} - 4\hat{j} - 3\hat{k}) = 8 \neq 0$ Hence lines will not intersect. So the lines are skew. Shortest Distance = $\frac{ (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) }{ \vec{b}_1 \times \vec{b}_2 }$ $= \frac{8}{\sqrt{4 + 16 + 9}} = \frac{8}{\sqrt{29}}$	$\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$ 1
OR		
Ans(b)	Let the wicket keeper divides the line segment in ratio $k : 1$ $\therefore \vec{W} = \frac{k\vec{F} + 1\vec{B}}{k + 1}$ $\Rightarrow 6\hat{i} + 12\hat{j} = \left(\frac{12k + 2}{k + 1}\right)\hat{i} + \left(\frac{18k + 8}{k + 1}\right)\hat{j}$ $\Rightarrow k = \frac{2}{3}$ Hence, the required ratio is 2 : 3 	1 1 1

Q31.	(a) The probability distribution for the number of students being absent in a class on a Saturday is as follows :											
	<table><tr><td>X</td><td>0</td><td>2</td><td>4</td><td>5</td></tr><tr><td>P(X)</td><td>p</td><td>2p</td><td>3p</td><td>p</td></tr></table>	X	0	2	4	5	P(X)	p	2p	3p	p	
	X	0	2	4	5							
	P(X)	p	2p	3p	p							
Where X is the number of students absent.												
(i) Calculate p.		1										
	(ii) Calculate the mean of the number of absent students on Saturday.	2										
	OR											
	(b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.											
Ans(a)	(i) Since $\sum P(X)=1 \Rightarrow p+2p+3p+p=1$ $\Rightarrow p=\frac{1}{7}$ (ii) Mean = $\sum X.P(X)=0(p)+2(2p)+4(3p)+5(p)$ $=21p=21\left(\frac{1}{7}\right)=3$	$\frac{1}{2}$ $\frac{1}{2}$ 1 1										
OR												
Ans(b)	Let E_1 : The applicant is a male E_2 : The applicant is a female A : The candidate chosen will have distinction in the written test. $P(E_1)=\frac{1}{3}, P(E_2)=\frac{2}{3}, P(A E_1)=0.4, P(A E_2)=0.35$ $\therefore P(A)=P(E_1)P(A E_1)+P(E_2)P(A E_2)$ $=\frac{1}{3}\times 0.4+\frac{2}{3}\times 0.35$ $=\frac{11}{30}$	$\frac{1}{2}$ 1 1 $\frac{1}{2}$										

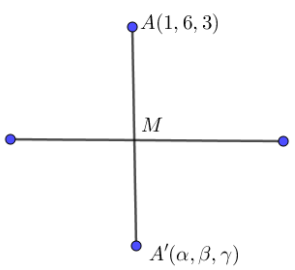
SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

Q32.	A school wants to allocate students into three clubs : Sports, Music and Drama, under following conditions : - The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club. - The number of students in Music club should be 20 more than half the number of students in Sports club. - The total number of students to be allocated in all three clubs are 180. Find the number of students allocated to different clubs, using matrix method.	
Ans	Let x, y and z be the no. of students allocated to Sports, Music and Drama clubs respectively. Here, $x = y + z, y = \frac{x}{2} + 20, x + y + z = 180$ $\Rightarrow x - y - z = 0, x - 2y = -40, x + y + z = 180$ Given equations can be written as $AX = B$ where, $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -2 & 0 \\ 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $A = -4 \neq 0 \Rightarrow A^{-1}$ exists. $adjA = \begin{bmatrix} -2 & 0 & -2 \\ -1 & 2 & -1 \\ 3 & -2 & -1 \end{bmatrix}$ $A^{-1} = \frac{1}{ A } \times adjA = \frac{1}{-4} \begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}$ $X = A^{-1}B$ $= \frac{1}{-4} \begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix} = \begin{bmatrix} 90 \\ 65 \\ 25 \end{bmatrix}$ $\therefore x = 90, y = 65, z = 25$ Number of students allocated in sports, music and drama are 90 , 65 and 25 respectively .	1½ ½ ½ 1 ½ 1

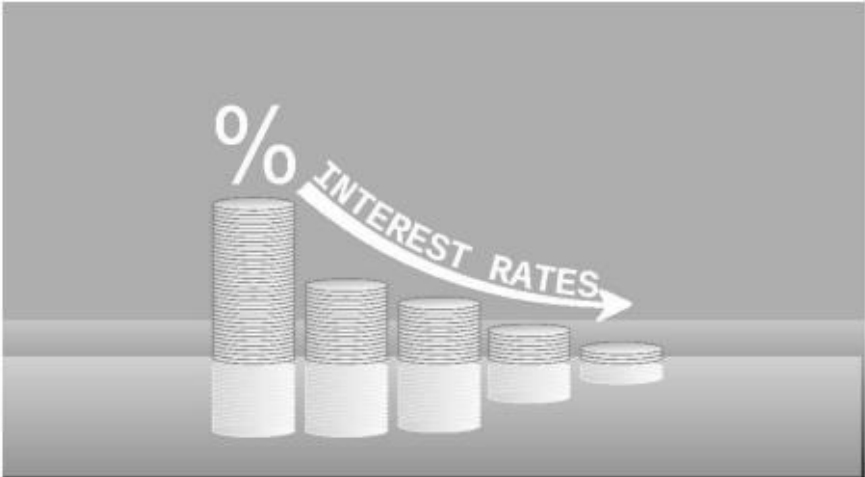
Q33.	Find : $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$.	
A33.	Put $x = a \tan^2 \theta$, $dx = 2a \tan \theta \sec^2 \theta d\theta$ $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx = 2a \int \sin^{-1} \sqrt{\frac{\tan^2 \theta}{1+\tan^2 \theta}} (\tan \theta \cdot \sec^2 \theta) d\theta$ $= 2a \int \theta \cdot (\tan \theta \sec^2 \theta) d\theta$ $= 2a \left[\theta \frac{(\tan \theta)^2}{2} - \frac{1}{2} \int (\tan \theta)^2 d\theta \right]$ $= a [\theta \tan^2 \theta - (\tan \theta - \theta)] + C$ $= a \left[\tan^{-1} \left(\sqrt{\frac{x}{a}} \right) \cdot \frac{x}{a} - \sqrt{\frac{x}{a}} + \tan^{-1} \left(\sqrt{\frac{x}{a}} \right) \right] + C$ or $(a+x) \tan^{-1} \left(\sqrt{\frac{x}{a}} \right) - \sqrt{ax} + C$	1 1 1 1 ½ ½
Q34.	(a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$. OR (b) If $x = a \left(\cos \theta + \log \tan \frac{\theta}{2} \right)$ and $y = \sin \theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.	
Ans(a)	Let $x = \sin A$, $y = \sin B \Rightarrow A = \sin^{-1} x$, $B = \sin^{-1} y$ $\therefore \sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ $\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$ $\Rightarrow 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) = 2a \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$ $\Rightarrow \cot \left(\frac{A-B}{2} \right) = a \Rightarrow A-B = 2 \cot^{-1} a$ $\Rightarrow \sin^{-1} x - \sin^{-1} y = 2 \cot^{-1} a$ differentiate both sides wrt x, $\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$	1 1 1 ½ 1½
OR		

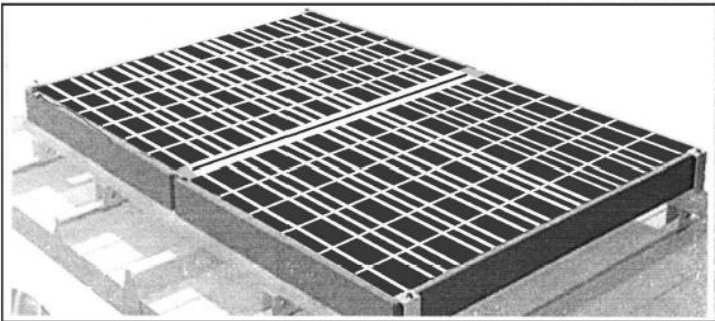
Ans(b)	$x = a \left(\cos \theta + \log \tan \frac{\theta}{2} \right)$ $\Rightarrow \frac{dx}{d\theta} = a \left(-\sin \theta + \frac{1}{\tan \frac{\theta}{2}} \times \sec^2 \frac{\theta}{2} \times \frac{1}{2} \right)$ $= a \left(-\sin \theta + \frac{1}{\sin \theta} \right) = a \left(\frac{1 - \sin^2 \theta}{\sin \theta} \right)$ $\frac{dx}{d\theta} = a \cot \theta \cos \theta$ Also, $y = \sin \theta \Rightarrow \frac{dy}{d\theta} = \cos \theta$ $\therefore \frac{dy}{dx} = \frac{\tan \theta}{a}$ Differentiating wrt x, $\frac{d^2 y}{dx^2} = \frac{\sec^2 \theta}{a} \times \frac{d\theta}{dx}$ $= \frac{\sec^3 \theta \tan \theta}{a^2}$ $\left[\frac{d^2 y}{dx^2} \right]_{\text{at } \theta = \frac{\pi}{4}} = \frac{2\sqrt{2}}{a^2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 1 1
Q35.	(a) Find the image A' of the point A(1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A'. OR (b) Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point Q(2, 4, -1) is 7 units. Also, find the equation of line joining P and Q.	

Ans(a)	The equation of given line is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$ Any arbitrary point on the line is $M(\lambda, 2\lambda + 1, 3\lambda + 2)$ dr's of AM are $\langle \lambda - 1, 2\lambda - 5, 3\lambda - 1 \rangle$ Here $1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$ $\Rightarrow \lambda = 1$ $\therefore M(1, 3, 5)$ is the foot perpendicular of the point A to the given line. Let image of point A in the line be $A'(\alpha, \beta, \gamma)$ Since M is the mid-point of AA', so $M\left(\frac{1+\alpha}{2}, \frac{6+\beta}{2}, \frac{3+\gamma}{2}\right) = M(1, 3, 5)$ $\Rightarrow A'(1, 0, 7)$ is the image of A. Also, Equation of AA' is $\frac{x-1}{0} = \frac{y-6}{-3} = \frac{z-3}{2}$	 1 1 $\frac{1}{2}$ 1 1
OR		
Ans(b)	The given line is $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$ and $Q(2, 4, -1)$ Any random point on the line will be given by $P(\lambda - 5, 4\lambda - 3, -9\lambda + 6)$ Since $PQ = 7 \Rightarrow \sqrt{(\lambda - 7)^2 + (4\lambda - 7)^2 + (-9\lambda + 7)^2} = 7$ $\Rightarrow 98(\lambda^2 - 2\lambda + 1) = 0 \Rightarrow \lambda = 1$ Hence, the required point is $P(-4, 1, -3)$ The equation of line PQ is $\frac{x+4}{6} = \frac{y-1}{3} = \frac{z+3}{2}$ or $\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$	 1 1 1 1 1

SECTION E

This section comprises 3 case study-based questions of 4 marks each.

Q36.	A class-room teacher is keen to assess the learning of her students the concept of “relations” taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$: $R_1 = \{(2, 3), (3, 2)\}$ $R_2 = \{(1, 2), (1, 3), (3, 2)\}$ $R_3 = \{(1, 2), (2, 1), (1, 1)\}$ $R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$ $R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$ The students are asked to answer the following questions about the above relations : (i) Identify the relation which is reflexive, transitive but not symmetric. (ii) Identify the relation which is reflexive and symmetric but not transitive. (iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive. OR (iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation ?	
Ans	(i) R_4 (ii) R_5 (iii)(a) R_1 and R_3 OR (iii)(b) Required pairs to be added to make the relation R_2 as an equivalence relation are: $(1,1), (2,2), (3,3), (2,1), (3,1)$ and $(2,3)$	1 1 1+1 2
Q37.		

	A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively. Based on the above information, answer the following : (i) What is the probability that a customer after availing the loan will default on the loan repayment ? 2 (ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest ? 2	
Ans	E_1 :customer avails loan on fixed rate E_2 :customer avails loan on floating rate E_3 :customer avails loan on variable rate A:the person defaults on the loan $P(E_1) = \frac{1}{10}, P(E_2) = \frac{2}{10}, P(E_3) = \frac{7}{10}$ $P(A E_1) = \frac{5}{100}, P(A E_2) = \frac{3}{100}, P(A E_3) = \frac{1}{100}$ (i) $P(A) = P(E_1) \cdot P(A E_1) + P(E_2) \cdot P(A E_2) + P(E_3) \cdot P(A E_3)$ $= \frac{1}{10} \times \frac{5}{100} + \frac{2}{10} \times \frac{3}{100} + \frac{7}{10} \times \frac{1}{100}$ $= \frac{18}{1000} \text{ or } \frac{9}{500}$ (ii) $P(E_3 A) = \frac{P(E_3) \cdot P(A E_3)}{P(E_1) \cdot P(A E_1) + P(E_2) \cdot P(A E_2) + P(E_3) \cdot P(A E_3)}$ $= \frac{\frac{7}{10} \times \frac{1}{100}}{\frac{18}{1000}}$ $= \frac{7}{18}$	1 1 1 1
Q38.	 A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.	

	Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres. Based on this information, answer the following questions : (i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y. 1 (ii) Write the area of the solar panel as a function of x. 1 (iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area. 2 OR (iii) (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition. 2	
Ans	(i) $2x + 3y = 300$ 1 (ii) $A = xy = \frac{x}{3}(300 - 2x)$ 1 (iii)(a) $A = \frac{x}{3}(300 - 2x) = \frac{1}{3}(300x - 2x^2)$ $\Rightarrow \frac{dA}{dx} = \frac{1}{3}(300 - 4x)$ ½ For critical points, put $\frac{dA}{dx} = 0 \Rightarrow x = 75$ ½ Also, $\frac{d^2A}{dx^2} = -\frac{4}{3} < 0$. So, A is maximum at $x = 75$ ½ Also, maximum area is $A = \frac{75}{3}(300 - 150) = 3750 \text{ m}^2$ ½ OR (iii)(b) $A = \frac{x}{3}(300 - 2x) = \frac{1}{3}(300x - 2x^2)$ $\Rightarrow \frac{dA}{dx} = \frac{1}{3}(300 - 4x)$ ½ For critical points, put $\frac{dA}{dx} = 0 \Rightarrow x = 75$ ½ As $\frac{dA}{dx}$ changes its sign from positive to negative as x passes through $x = 75$ from left to right, which means $x = 75$ is the point of maximum. ½ Also, maximum area is $A = \frac{75}{3}(300 - 150) = 3750 \text{ m}^2$ ½ Note : Full credit to be given if the student takes equation as $2x + 2y = 300$ or $2x + 4y = 300$ or $4x + 4y = 300$ or $4x + 3y = 300$ The solutions of sub-parts will differ and marks may be given accordingly.	